Multidisciplinary, Multilingual, Multilevel and Multipurpose Usage of GeoGebra Software in Education

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Abstract. The technical characteristics of GeoGebra have been examined, its various views and tools have been described, with special emphasis on interactive tools. The use of GeoGebra in education has been depicted, mainly in mathematics. We emphasized the importance of sharing GeoGebra digital learning materials within the GeoGebra community, which is present and active in more than 190 countries now and is growing astonishingly fast. In this context, various forms of organizing GeoGebra digital learning materials are reviewed: GeoGebra Materials, GeoGebra Wiki, GeoGebra Tube, GeoGebra Books and GeoGebra Exercises.

The author demonstrates the universal and effective application of the GeoGebra software in four dimensions. First, GeoGebra can be applied in a variety of disciplines, primarily in mathematics and computer science, and in subjects in the field of natural and technical sciences. Second, the use of GeoGebra also covers a wide range of applications, from primary education to higher education. Third, GeoGebra enables the creation of multilingual digital learning content. Finally, various variations of animations and simulations with different weights can be made in GeoGebra, which can enhance the individualization of teaching within multiple levels of education and enable conditions for programmed instruction.

A detailed overview of three conducted researches and the obtained results are provided. The researches include filling in questionnaires by teachers and students, as well as performing teaching lessons in Mathematics and Physics in two secondary schools with students divided into experimental groups (where GeoGebra animations and simulations were applied) and control groups (where the classes were held in classical, traditional way, without the use of educational software).

According to our researches and observations, free and open-source software like GeoGebra is a great opportunity that should be used to the fullest extent in our education especially in the conditions of obvious need for educational software and digital educational materials, corresponding to our education curricula. In the concluding observations, we give concrete conclusions, suggestions and recommendations for implementing GeoGebra in the educational system.

Keywords: GeoGebra, application of GeoGebra in education, free and open educational software.

1. Introduction

GeoGebra is a professionally developed, freely accessible, interactive and dynamic educational mathematical open source software for teaching and learning mathematics at all levels of education, from elementary school to university (GeoGebra, 2019), (Chrysanthou, 2008), (Kllogjeri and Kllogjeri, 2014). It is written in Java programming language, and its web application in HTML5. GeoGebra derives its name from the fact that it unites mathematical disciplines algebra and geometry (Mukiri, 2016). The creator of GeoGebra is Markus Hohenwarter, who first introduced this software in his master's thesis in the field of mathematics and computer science, defended at the University of Salzburg, Austria in 2002 (Hohenwarter and Lavicza, 2010), (Mukiri, 2016). Since 2006, the GeoGebra project has enjoyed the support of the Ministry of Education of Austria, which provides access to GeoGebra for mathematics education in schools and universities around the globe.

Since July 2006, the development of GeoGebra continues in the United States at the University of Florida Atlantic, where it is actively used in regular classes. (Mukiri, 2016)

GeoGebra program is a free program (GNU license) used worldwide. It is available in more than 64 world languages (Hohenwarter and Lavicza, 2010) which allows the use of local language software and multicultural learning environments (Chrysanthou, 2008). In many educational systems around the world, GeoGebra is integrated into textbooks and various projects.

GeoGebra has received more prestigious international awards for the best educational software, including: Archimedes 2016, MERLOT Classics Award 2013, NTLC Award 2010, Tech Award 2009, BETT Award 2009, SourceForge.net Community Choice Awards 2008, AECT Distinguished Development Award 2008, Learnie Award 2006, eTwinning Award 2006, Trophées du Libre 2005, Comenius 2004, Learnie Award 2005, Digita 2004, Learnie Award 2003, EASA 2002. (GeoGebra, 2019), (Hohenwarter and Lavicza, 2010).

The GeoGebra community exists in almost every country in the world. It is constantly evolving and has millions of users.

2. Technical Characteristics of GeoGebra

2.1. *Basic Information*

GeoGebra is designed to combine the properties of dynamic geometric software (for example, Cabri Geometry, Geometer's Sketchpad, Eucklides, Geonext, Descartes, Cinderella, EucliDraw, and others) and a computer algebra system (for example, Derive, Maple) in a single, integrated and easy-to-use system for teaching and learning mathematics (Hohenwarter & Preiner, 2007b) (Hohenwarter and Lavicza, 2010). GeoGebra has the ability to bridge the differences that exist between mathematical

disciplines, especially between geometry, algebra, and mathematical analysis (Kllogjeri and Kllogjeri, 2014).

The latest stable version of GeoGebra is the version 6.0.523.0 from January 31, 2019. It can be used on several operating systems: Windows, MacOS, Debian, Ubuntu, Red Hat, Linux, openSuse, Android, iOS, but also as a web application.

Although though GeoGebra possesses powerful features, it does not have large system requirements. It can be installed on all operating systems and is really straightforward. GeoGebra's virtual tools can be easily used, and the GeoGebra environment can be very cozy and attractive because the work with it resembles playing a computer game (Kllogjeri and Kllogjeri, 2014).

GeoGebra's graphics view is of high quality, and the drawings made by GeoGebra can be easily transferred to other presentations and programs (for example, Latex). GeoGebra can be used to create interactive tutorials, animations and website simulations (GeoGebra, 2019). GeoGebra's user interface is flexible and can be tailored to the needs of students. According to Hohenwarter and Preiner (2007), GeoGebra is a friendly software that can be used intuitively and does not require advanced skills. (Kllogjeri and Kllogjeri, 2014)

2.2. *Description of GeoGebra views and Toolbar*

GeoGebra's interface includes: a menu, a toolbar, Algebra view (or window), Graphics view (window), Spreadsheet view (window), and Input bar (Fig. 1).
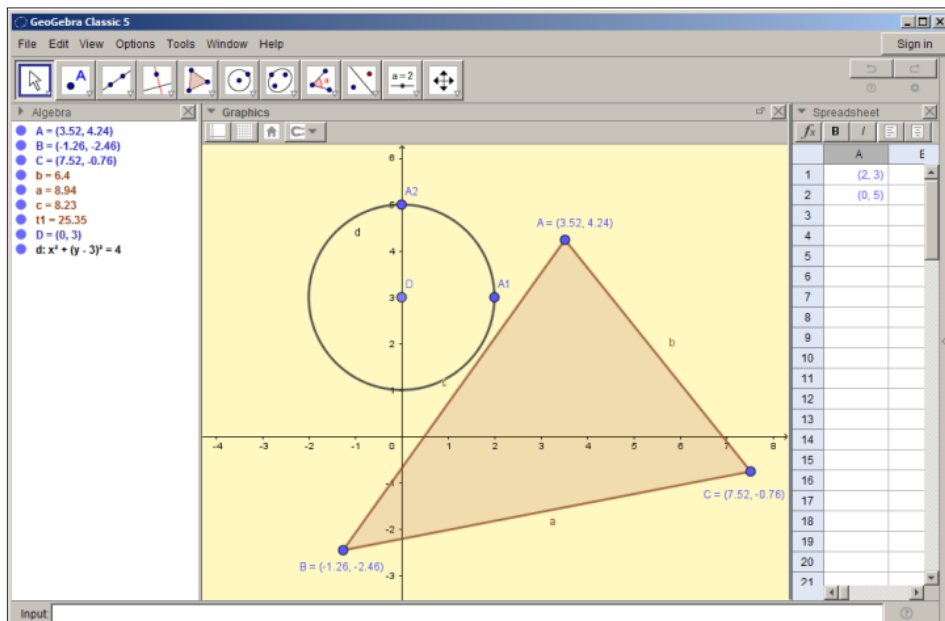

Fig. 1. Three GeoGebra views (windows).

By using the Input bar and the Enter key, the following data can be directly entered in the Algebra view of GeoGebra: constant values and parameters, coordinates of points and vectors, complex numbers, mathematical expressions (arithmetic, algebraic, trigonometric, analytical), equations and inequalities (algebraic, transcendent, differential, integral), systems of equations and inequalities, commands and functions with one or more variables (linear, square, cubic, exponential, logarithmic, and others). A more detailed perspective of the Algebra view is the Construction protocol. It describes all steps of the interaction between the user and the computer.

Using the virtual tools from the GeoGebra toolbar, a variety of geometric objects can be constructed in the Graphics view, including: points, vectors, angles, lines, line segments, series of line segments, polygons, graphs of functions, curves, conic sections, and other more complex objects. In this view, the coordinate axes and the coordinate grid can be turned on/off, and text can also be entered.

Using the spreadsheet, numerical data can be entered in the worksheet. They can later be used for creating lists of values, lists of points and matrices. Also, they can be applied for statistical data processing, hypotheses testing for particular distributions, etc.

We concluded that there are three ways of displaying the objects: Algebra view, Graphics view and Spreadsheet view. If we create (construct) a geometric object in the Graphics view through an appropriate geometric tool, then the corresponding algebraic representation (mathematical expression, command and function) of that object will automatically be created in the Algebra view. On the other hand, after each direct input (via a keyboard) of a mathematical expression, command, or function in the Input bar, an appropriate geometric object is automatically created in the Graphics view. Thus, the object (objects) in the Graphics view corresponds to the expression (expressions) in the Algebra view and vice versa. Any change (transformation) of an object in the Graphics view automatically causes an appropriate change of algebraic properties of that object, which leads to an appropriate alteration of the Algebra view content. The reverse is also true. So, all displays of an object are dynamically connected and, regardless of the way the object was created, any change in any of its views (displays) causes changes in other views (displays). Multiple representations of objects are one of the greatest advantages of GeoGebra.

2.3. *Interactive Tools in GeoGebra*

In order to include interactivity in the GeoGebra documents, we can use the following types of objects, included in the toolbox:

1. Check boxes; they represent graphical representations of '0' and '1'.
2. Input boxes; they have the same function as text input for scripts. The scripts are activated in the following three cases: when the text is changed in the Input box, when the Enter key is pressed or when the Input box is abandoned.
3. Buttons; clicking a certain button activates its corresponding script. Although scripts can also be activated by clicking on other objects, such as images, the use of buttons makes the applications more intuitive.

4. Dropdown lists; they are used to display the content of already created lists in the form of a drop-down menu. To create a drop-down list, the user needs to check the "Draw as drop-down list" field on the "Basics" tab of the "Object Properties" dialog box. It is activated from the context menu which is generated by right-clicking on the appropriate list (the list which is intended to be displayed in the form of a drop-down menu). Drop-down lists can also be created with appropriate commands.

The first three types of objects can be formed in two ways, through the appropriate tool from the toolbox or by entering the corresponding command in the Input bar.

3. International GeoGebra Cooperation and Open Educational Resources

3.1. *International GeoGebra Cooperation*

Over the past few decades, it has been shown that a group of enthusiasts can change people's conventional thinking and patterns for development and innovation. The success of open source projects such as Linux, Firefox, Android, MOODLE and Wikipedia have shown that collaboration and sharing can produce valuable resources in different areas of life. Hohenwarter and Lavicza (Hohenwarter and Lavicza, 2010) emphasize the fact that the international GeoGebra community raised the awareness about the importance of GeoGebra.

GeoGebra is a fast growing community with over 50 million users from around 190 countries worldwide (Stephen Jull's LinkedIn page, 2018). GeoGebra's open source code encourages communication and collaboration among its users across the world. Users can contribute with their own creations or produce customized versions of existing worksheets. Questions and ideas about GeoGebra can be discussed in GeoGebra's user forum (GeoGebra User Forum, 2019).

3.2. *International GeoGebra Institute*

At the end of 2007, the International GeoGebra Institute (IGI) was established to support GeoGebra community members and teachers who were starting to use GeoGebra by offering a community forum that will expand cooperation and interconnections. (Hohenwarter and Lavicza, 2010)

The International GeoGebra Institute is the main umbrella organization of local GeoGebra institutes founded by university teachers and researchers and teacher education institutions (Hohenwarter & Lavicza, 2007). Four main objectives of the International GeoGebra Institute are: trainings and support for teachers, preparation of teaching materials and software, research and coverage of less developed communities. The local GeoGebra institutes follow the goals of the international GeoGebra Institute, but the emphasis on their work depends on their local needs, interests and priorities. (Hohenwarter, Lavicza)

There are currently 132 GeoGebra Institutes in 58 countries in the world (GeoGebra Institutes, 2019).

3.3. *GeoGebra Materials*

GeoGebra's users (teachers, students, researchers, software developers and other enthusiasts) can create original interactive dynamic teaching materials (interactive GeoGebra websites, applets, constructions, and other learning resources) that can then be used and shared online via a platform called GeoGebra Materials. It contains over 170.000 materials and is constantly increasing on a daily basis. Users can also take an advantage of the software to solve open problems. (Stephen Jull's LinkedIn page, 2018)

The GeoGebraWiki pool (GeoGebra Wiki Pool, 2019) appeared shortly after the appearance of GeoGebra. In 2011, the successor to the GeoGebra Wiki Platform, also known as GeoGebraTube, appears, in line with the role it has, since it practically represented YouTube for free GeoGebra materials. The GeoGebraTube platform was renamed in 2016 to GeoGebra Materials. GeoGebra Materials can also be defined as an official cloud service and a repository of interactive resources for learning and teaching. This service has over million resources, over 40% of which are publicly shared as searchable materials – such as interactive worksheets, simulations, games and e-books created through the GeoGebraBook. In each book's chapter, worksheets can be stored. Through simple metadata, you can easily search for existing materials regarding different levels, themes, and languages.

GeoGebra materials can be exported in multiple formats, such as static images or animations. Issues of organizing and ensuring the quality of resources, as well as linking them to other open educational resources platforms, are still open.

3.4. *GeoGebra Exercises*

Interactive applets ("Mathlets") can be created in the framework of the GeoGebra Exercises project, which will generate random questions to suit students. They provide feedback to the students (visual or textual). The response checks are done with a combination of JavaScript and GeoGebra (including the GeoGebra symbolic algebraic system).

These interactive applets can then be imported as a SCORM package in a learning management system, for example MOODLE, and the results will be recorded. The level can range from primary (e.g, fractions) to advanced (for instance, mathematical analysis), and the goal is to fully cover the entire high school curriculum. (Hohenwarter&Lavicza)

First, GeoGebra can be applied in a variety of disciplines, primarily in mathematics and computer science, and in subjects in the field of natural and technical sciences. Second, the use of GeoGebra covers a wide range of applications, from primary education to higher education. Third, GeoGebra enables the creation of multilingual digital learning content. A wide range of animations and simulations with different difficulty levels can be made in GeoGebra, which can enhance the individualization of teaching within multiple levels of education and enable conditions for programmed instruction.

4. The Application of GeoGebra in Education

4.1. *GeoGebra and Education*

GeoGebra is created for educational purposes. It combines contents of the fields of geometry, algebra, statistics, analysis, and spreadsheets and graphics in a simple, easy-to-use package (GeoGebra, 2019), (Leggett, 2014). GeoGebra is a virtual experimental laboratory which can be used: as a place where hypotheses can be posed and tested; to monitor, verify and confirm scientific facts; to illustrate tasks and problems, their variations and sets of solutions; to analyze the properties of different geometric objects; to discover geometric points of points that satisfy certain conditions, to find important points of functions; to prove mathematical statements; to demonstrate various evidence of mathematical statements and the like.

We took particular care to ensure that the content and weight of digital educational materials included in different levels of education correspond to the age of students and can be adapted to the curricula of the respective subjects. It is important to note that where it was allowed, a part of the digital learning materials created for a particular subject, for example Mathematics for primary education, were used as a basis for the development of appropriate digital learning materials for Mathematics for secondary education, with carefully adjusted degree of complexity. The same was true in reverse, applying the appropriate simplification.

Also, almost any digital content can serve as a basis for creating other digital content. Namely, the created simulations include several parameters that can be manipulated by the student. The involvement of only one parameter means that the simulation includes an entire family of tasks, situations, and events corresponding to that parameter.

GeoGebra is equally popular among students and teachers. Students are attracted primarily to its dynamism and interactivity, but also to its ability to provide a visual and conceptual feedback. It has been acknowledged that GeoGebra encourages students to learn mathematics (Hohenwarter and Preiner, 2007), helps them to analyze various problems and to simulate physical phenomena through dynamic structures. It offers unlimited experimental opportunities to students, which leads to an improved content understanding and more effective learning. GeoGebra is a free software and students can use it not only at school, but also at home, while doing homeworks, practicing, revising the courses' material and preparing for future lessons. (Chrysanthou, 2008)

The most common application of GeoGebra is in the field of teaching and learning mathematics. It can simultaneously serve as a tool for demonstrating and visualizing formal mathematical knowledge, authoring, creating interactive learning materials and facilitating collaboration and communication. Except as a computer tool, many educators consider GeoGebra a conceptual, pedagogical, cognitive and transformational tool in teaching and learning mathematics. This highlights the universality of GeoGebra in teaching mathematics and educational reforms in the field of mathematics. (Chrysanthou, 2008), (Bu and Schoen, 2011)

The flexibility of GeoGebra is multifunctional, it can be used in teaching Mathematics at all levels of education and in a wide range, from simple to complex structures. It is suitable for students with varying degrees of ability and can be used to master teaching content in other subjects (Physics, Chemistry, Biology, Astronomy, etc.). The GeoGebra website consideres GeoGebra a supporter of education in natural sciences, technology, engineering and mathematics (STEM) and innovation in teaching and learning around the world. According to Chrysanthou (Chrysanthou, 2008), GeoGebra promotes mathematical research and can enable effective application of constructive, cognitive and collaborative learning models in educational institutions at different levels. The extent of teachers who use GeoGebra is also wide, ranging from the preparation of teaching materials to making tests for students' knowledge assessment.

4.2. *GeoGebra in Computer Science Education*

Students often times have troubles when they are assigned to write a computer program. Throughout an extensive experience with students in the Computer Science field, we have detected that the main problems lie in their lack of understanding or misunderstanding about what has been assigned to them and how certain algorithms function. Mainly, students are given an initial problem and appropriate input data and they are asked to solve it using an effective algorithm. If students do not properly understand the question or the algorithm, they will not be able to cope with it in the correct way. Appropriately created and adapted animations, simulations and computer games portraying visual representation of algorithms can certainly contribute to students' better understanding about the assigned problems. GeoGebra does exactly this; its interactive nature helps students easily submerge into the secrets of computer programming and algorithms. One such example of a GeoGebra animation is illustrated in Fig. 2 where different sorting algorithms (Selection sort, Insertion sort, Merge sort, Bubble sort and Quick sort) are applied. By selecting any of the sorting algorithms, students can experiment with different sets of data input. This will not only show the correct solution (a sorted array), but it will show all its steps as well. Due to GeoGebra's visualization features, particularly the red and green colors applied to the numbers in this example, students can observe the actual comparison and predict the shifting of the numbers. Additionally, students can choose the next step button (step-by-step manner) or the animation button (all steps, one after another) with their own assigned pace.

Such an example solves the above-mentioned problem regarding the appropriate comprehension of the problem or algorithm. This problem solving approach will engage students from all levels, starting from young age to upper-class students, but will especially help the ones that have troubles with the school's material. GeoGebra will intellectually stimulate them to draw their own patterns about the work of certain algorithms and help them create a computer program with the correct solution of the problem. For the talented students who aim for national and international informatics Olympiads, GeoGebra offers programming in Java Script and GeoGebra Script. By using this framework, GeoGebra can effectively be applied in all subfields of Computer Science and
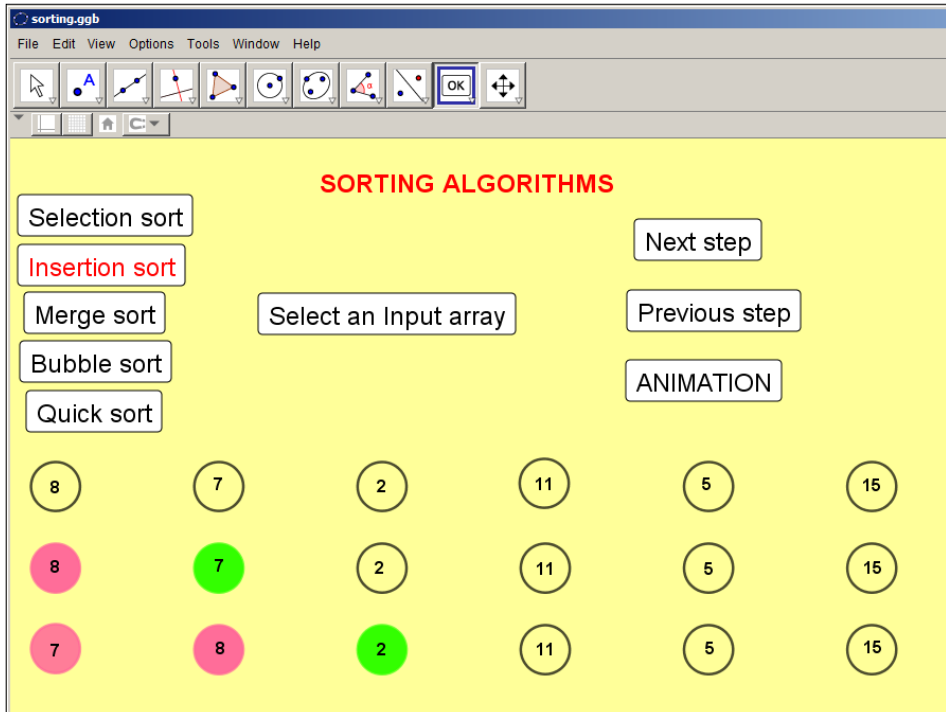
Fig. 2. GeoGebra animation for sorting algorithms.

Informatics to familiarize students with programming concepts, develop their computational thinking and facilitate their participation in competitions.

4.3. *Examples of GeoGebra Applications*

4.3.1. *Horizontally Launched Projectile Simulation*

Fig. 3 shows a GeoGebra simulation for a horizontally launched projectile, in this case, an apple. In this simulation, we practically demonstrate the multilingualism, an important feature that GeoGebra offers. Namely, the upper left corner of the Graphics view contains the buttons "MK", "EN" and "RU" that symbolize the three languages, Macedonian, English and Russian, respectively. An active language indication is the red lettering label. The number of languages we can include in the simulation is unlimited.

Three sliders were used, for initial speed (v_0), initial height (h) and the time of flight (t) of the projectile. At whatever moment of the apple movement, the Graphics view displays the current coordinates of the apple. According to the values of the sliders shown in the Fig. 3, it is clear that the initial position of the apple has coordinates (0, 7.5), and the initial speed of the apple is $v_0 = 10$. The Fig. 3 also displays the coordinates of the apple (4.9, 6.3) after t = 0.5 s and its current horizontal distance D = 4.9 m. At whatever moment of the movement, the coordinates of the apple satisfy the formulas displayed on
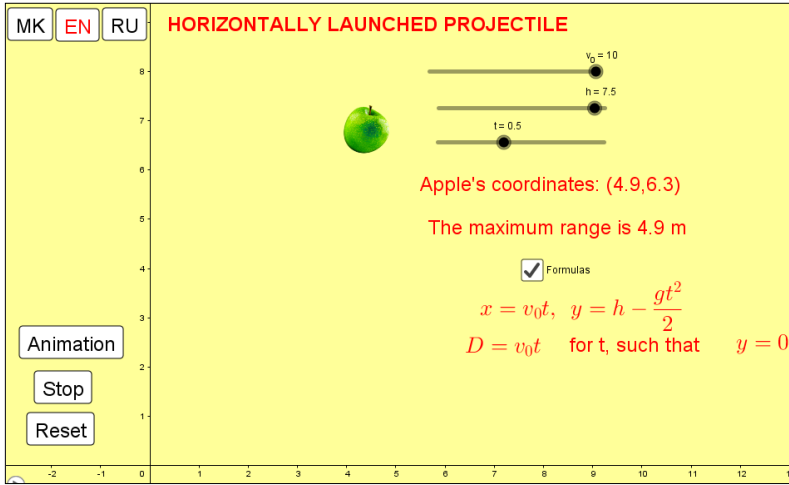
Fig. 3. GeoGebra simulation for horizontally launched projectile (English language version).

the lower-right part of the Graphics view. The "Animation", "Stop" and "Reset" buttons control the apple's motion in the projected path. Their meanings are "start movement", "stop movement" and "set the apple at the starting position", respectively.

4.3.2. *Synchronization of Analog and Digital Settings Application*

A little more sophisticated application is the "Synchonization of analog and digital settings" application (Fig. 4), which is intended for students in the lower grades in elementary schools.

Its elements are an analog clock on the left side, two check boxes ("Random Digital Time" and "Set Digital Time") on the right side and two buttons ("Generate random analog time" and "Set the arrows") below the analog clock.
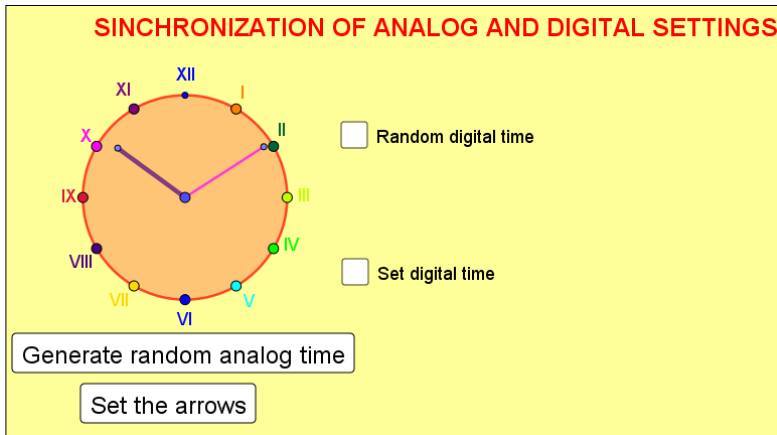

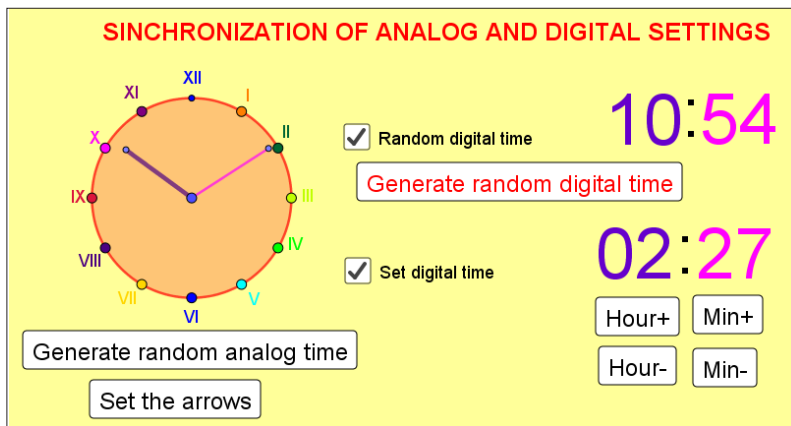Fig. 4. Synchronization of analog and digital settings (version 1).

Fig. 5. Synchronization of analog and digital settings (version 2).

If we click on the first button, random time is automatically generated. More precisely, in the background of the application, random values from the interval (0.2π) for both the hour hand angle of rotation and the minute hand angle of rotation are selected. The center of rotation of the hour and the minute hands is the center of the clock (the point where the beginning of the two hands is fixed). Clicking the second button releases both the hour and the minute hand and they can be moved manually.

We can move the hands by clicking and dragging their ends, i.e. by rotating the hands about the center point of the clock. After clicking ("ticking") on each of the "Random Digital Time" and "Set Digital Time" fields, two digital clocks are generated right next to them (Fig. 5).

Unlike the case with the analog clock, after clicking the "Generate random digital time" button, we generate four random numbers h1, h2, m1 and m2, which represent the time h1h2:m1m2. The first two are the hour digits, and the second two are the minute digits. When the "Set Digital Time" checkbox is "checked", it displays a digital time that the user can manually adjust by clicking on the the following buttons: "Hour+","Hour-", "Min+" and "Min-".

Thanks to the flexibility of GeoGebra, this application can serve as a basis for creating other applications. By combining the existing parameters (buttons, checkboxes), excluding some of them and/or including new parameters, we can create multiple applications, animations and simulations with different weights. The teachers who are familiar with these GeoGebra capabilities will be able to adjust the degree of application complexity and to enhance the individualization of teaching and learning.

There are lot of possible modifications of this application. We can set the clocks (analog and digital) to display the same time. Then, the speed of the animation can be adjusted to be equal to the speed of displaying the exact time of a real clock. In this way, the animation could serve as a classic clock. In another modification of this application, both clocks could serve as stopwatches. Possible upgrades to this modification include activating an alarm at a predefined time together with a sound file. To increase reality of

all possible versions of this application, we can activate an appropriate sound file in the background that will simulate the clock ticking sound effect.

Teachers can anticipate a full range of tasks related to the basic version of this application. Below is a list of possible general problem tasks that a teacher can set up:

i)   Generate a random analog time, then match the digital setting to the analog one.
ii) Generate random digital time, then match the analog setting to the digital one.
iii) Generate a random analogue time, then adjust the digital setting to be forward or backward for a certain number of hours and/or minutes in relation with the analog setting.
iv) Generate a random digital time, then adjust the analog setting to be forward or backward for a certain number of hours and/or minutes in relation with the digital setting.

5. The GeoGebra Researches

In Fall 2018, we conducted three researches. The first research targeted primary and high school teachers regarding their knowledge, the frequency, the means and the purposes of their GeoGebra use during classes. The second research focused on first-year university students' answers to questions related to their GeoGebra use during their primary and high school years. The third research examined the effectiveness of GeoGebra's use through a comparison between two types of class groups, experimental (with use of GeoGebra) and control (without use of ICT) group. Through these researches we got a clear image of GeoGebra's use in schools, particularly the profile of GeoGebra's users and their dedication for using GeoGebra. We investigated the amount of knowledge users (students and teachers) have about GeoGebra through the method of self-evaluation and the extent to which GeoGebra animations, simulations and computer games can contribute to a better understanding of the learning material.

5.1. *The First Research*

In our first countrywide research, 207 teachers from 27 cities participated. The electronic questionnaire was created in Google Form and included 36 questions.

The allocation of teachers by subjects they teach is as follows: Mathematics (38%), Informatics (26.5%), Natural sciences (19%), Programming languages (17%), Work with computers and basics of programming (5.5%), Information technology (13%), Technical education (10.5%), Physics (9.5%), Biology (8%), Chemistry (7.5%), Electrical engineering (6.5%), Linear algebra and Analytic geometry %) Algebra (5%), Mathematical analysis (5%), and Geography (3%).

52.7% of total number of responders are secondary education teachers, 19.5% are lower primary school teachers, and 27.8% are upper primary education teachers.

Over 70% of the respondents have many years of experience in teaching (38.2% of them have more than 20 years of experience, 31.9% have 11 to 20 years of experience,

19.1% have 6 to 10 years of experience, 8.3% 2 to 5 years, and only 2.5% of the respondents have less than 2 years of experience).

The results of the teachers' self-evaluations indicated that 55.9% of them do not have any knowledge of this software. 20.4% of the respondents have a basic level of GeoGebra knowledge, 18.8% have medium level of knowledge, and only 4.8% of the respondents answered that they have advanced level of GeoGebra knowledge.

The results of the research also showed that GeoGebra is most often used for drawing function graphics (86.2%), for solving constructive tasks (48.3%), for displaying animations (48.3%), for displaying simulations (41.4%), for displaying geometric bodies and/ or functions in three-dimensional space (25.9%) and for solving different types of equations (24.1%). From these responses and from the answers to the other questions, it can be concluded that teachers who use GeoGebra in schools, use only few opportunities offered by this software.

Approximately one quarter (24.5%) of the respondents do not use animations, simulations and/or computer games in the process of teaching, 32.8% of the respondents use them once or twice a month, 27% once to twice a week, and only 7.4% use them daily.

5.2. *The Second Research*

This research was conducted over a sample of 187 first grade university students who attend the bachelor program "Application of Information Technologies" at the Faculty of Information Sciences and Computer Engineering at the University "St. Cyril and Methodius" in Skopje. They had obtained a degree in secondary education in a total of 32 cities. The 19-questions questionnaire was filled in by the respondents on paper. The majority of the respondents, i.e. 67.9% completed high school education (Gymnasium), 20.9% of them secondary vocational education, and 9.1% of the respondents obtained a diploma from a secondary economic school.

About 72% of them stated that they did not use the GeoGebra software during their primary education, and over 56% did not use it during the secondary education. Out of all students who used GeoGebra software, 10.4% self-assessed themselves with the lowest grade "1", 23.5% with grade "2", 42.6% with grade "3", 20% with grade "4", and only 3.5% with grade "5".

5.3. *The Third Research*

The third research consisted of three parts. The first part consists of an experiment conducted in two Physics classes of first year high school students (one experimental and one control group). The second part consists of an experiment conducted in two Physics classes of third year students (one experimental and one control group) in the same high school as the first part. The third part consists of an experiment conducted in two Mathematics classes of fourth-year students (one experimental and one control group) in another high school. The experimental groups used GeoGebra simulations and the control groups had instruction conducted without the use of ICT tools.

Table 1

The main features of the third research

	First part	Second part	Third part
School title City	Orce Nikolov Skopje	Orce Nikolov Skopje	Josif Josifovski Gevgelija
Subject	Physics	Physics	Mathematics
High school year	First	Third	Fourth
Lesson title	Horizontally launched projectile	Polarization of light	Real functions
Total number of students	68	59	42
Number of students in experimental class	34	29	22
Number of students in control class	34	30	20
Simulation used (in experimental group)	Horizontally launched projectile	Partial and full polarization of the light (air-water case and air-glass case) and the Malus's law for light in-tensity	Graphics of: Square function, Sine function, Cosine function, Exponential fun-ction and a Logarithmic function

Afterwards, all experimental and control groups took a test (post-test) of the material from the previous lesson learned. In the end, we compared the results of students' previous test (pre-test) and the results achieved at the test (post-test) in both classes, in each of the three parts of the research. We took into account only the answers to the questions (tasks) closely related to the simulations used.

The results of the researches conducted using GeoGebra simulations, among other things, showed: a) identical average grades (4.05) from the pre- tests among students from both groups; b) a better average post-test grade in experimental groups of students (3.4 versus 3.26 in control groups), which was especially expressed in the results of the mathematics test; and c) the t-test was used during statistical processing of the data. We were considering Null Hypothesis H_0: $\mu_1 = \mu_2$ versus the Alternative Hypothesis H_1: $\mu_1 > \mu_2$. With a significance level of 0.0168, the average pre-test grade was better among the students from the experimental group, and with a level of significance 0.001 the same applies to the second test, whereby we conclude better success among the students who followed the instruction using the GeoGebra simulations. It is interesting to know that the statistical data processing was carried out by using the GeoGebra's tools.

6. General Conclusions and Recommendations

Taking into consideration all previous sections in this paper, if GeoGebra begins to be applied at all levels of education, there will be multiple benefits. First of all, it will modernize teaching, stimulate the creativity of students and their critical thinking skills, and will lead to easier adoption of 21st century skills among the students. Using GeoGebra in teaching will intensify the application of active learning methods,

such as Problem-based learning, Project-based learning, Learning with experimentation, Collaborative learning, etc. Secondly, students who start using GeoGebra at the beginning of their education will upgrade their knowledge more easily in the further levels of their education. Thirdly, if the GeoGebra software is installed in the schools, the problems of installation, use and upgrading that are inherent in many modern software tools will be minimized. This conclusion relies on the fact that GeoGebra does not have high technical requirements for the necessary hardware and software, it is easily installed on older computers, and even the most complex animations, simulations and computer games made in GeoGebra occupy minimum disk space, expressed in kilobytes. The use of GeoGebra will overcome the gap between opportunities provided by schools in rural and urban environments, as well as between well and poorly equipped schools. New possibilities for experimentation in teaching and learning will be opened, without the danger of having physical and health consequences, in comparison with the existence of classical laboratory experiments, which are available in a limited number of schools. In the our opinion, GeoGebra is mostly beneficial to gifted and talented students, the unjustly neglected category of students in our education, who can play an important role in the development of our society. Usually, teaching is tailored to the level of an average student and gifted students do not receive the necessary attention and care from their teachers, thus the children's further education is left to the enthusiasm, motivation and knowledge of those teachers. With the help of GeoGebra, all students will be given the opportunity to explore the incredible field of experimentation. GeoGebra can also be used as a tool for preparing students for competitions in different fields.

The researches conducted suggest that there is a room for improvement regarding teachers' and students' knowledge of this software. The results of the experiments particularly show that these users do not make the maximum out of GeoGebra's features. A peripheral area that was investigated during the first research was exactly the teachers' willingness to improve their GeoGebra skills. 69.4% of all teachers who took a part of the first research expressed their interest in attending events and trainings which can further contribute to expanding their knowledge for GeoGebra. As a result of that, we would get more teachers knowledgeable in this field, thus students will be given a better opportunity to use GeoGebra animations and simulations frequently under their teachers' supervision. The fact that students from the third research who use GeoGebra achieve better results compared to students who do not have this habit shows that GeoGebra is a highly promising software whose potential can further be used by qualified teachers and motivated students.

The proposal for the selection of GeoGebra as a high-quality software solution that is recognized, accepted and used worldwide, at the same time satisfies two important criteria that are of great importance in societies (developing countries) like ours, Namely, minimizing the costs and maximizing the quality. This proposal arises as a result of a part of the research and it relies on the commonly known benefits of the application of GeoGebra. GeoGebra is a free and open-source software tool and represent the ideal solution for our educational institutions, for students and their teachers.

References

Bu L., Schoen R., (2011). *Model-Centered Learning: Pathways to Mathematical Understanding Using GeoGebra,* Sense Publishers, Rotterdam.

Chrysanthou I. (2008). *The use of ICT in Primary Mathematics in Cyprus: The case of GeoGebra*, University of Cambridge, Faculty of Education, Cambridge.

GeoGebra (2019). `https://www.GeoGebra.org`

GeoGebra Institutes (2019). `https://wiki.GeoGebra.org/en/Category:GeoGebra_Institute`

GeoGebra s (2019). `https://help.GeoGebra.org/topics`

GeoGebra Wiki Pool (2019). `https://archive.GeoGebra.org/en/wiki/index.php/ Main_Page`

Hohenwarter, M., Lavicza, Z. (2ti010). *Gaining Momentum: GeoGebra Inspires Educators and Students.*

Jancheski, M. (2019). Educational software, digital learning materials, and teaching and learning by using ICT under conditions of mass informatization in education, Skopje.

Jancheski, M. (2019). Educational software for students from kindergartens and lower primary school. In: *13th International Techology, Education and Development Conference, Valencia, Spain. Proceedings*. Published in INTED2019. 7603–7612

Jancheski, M. (2019). GeoGebra animations, simulations, and computer games in teaching and learning science. In: *13th International Techology, Education and Development Conference, Valencia, Spain. Proceedings*. Published in INTED2019. 7613–7623

Kllogjeri, P., Kllogjeri, Q. (2014). *GeoGebra: A vital bridge linking mathematics with other sciences (available to everyone)*.

Leggett, A. (2014). *Active learning pedagogies: Problem-based learning.*

Mukiri, M.I. (2016). *Feasibility of Using GeoGebra in Teaching and Learning*. Department of Educational Communication and Technology, Kenyata University.

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