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Learning Algorithms with Unified and Interactive Web-Based Visualization

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Abstract. We present a *unified* and *interactive* web-based visualization of various classical and non-classical algorithms at
`http://www.comp.nus.edu.sg/~stevenha/visualization`.

Our collection of algorithm visualizations has the following advantages over many other web-based algorithm visualizations in the Internet – (1) it has visualizations of various non-classical algorithms that currently cannot be found elsewhere in the Internet; (2) it is interactive; users (usually students) can enter *their own* input data to test the behavior of the algorithm; (3) it has a *consistent* user interface across different algorithm visualizations that are currently available; (4) it is built with HTML5 making it accessible on modern portable PCs including tablets and smartphones. User studies in two algorithm classes in NUS show that our visualizations is of immense help to *some* students who prefer to learn *visually*.

Key words: unified, interactive, algorithm, visualization.

1. Introduction

To teach *data structures and algorithms* (henceforth referred to as *algorithms*) in a typical Computer Science class, a professor/lecturer/instructor/teacher (henceforth referred to as *teacher*) has to illustrate at least one (and preferably several) working example(s) on how a particular algorithm works. The typical teaching methods employed are as follows:

1. The examples are pre-scripted in printed handouts, textbooks, or in PowerPoint (or equivalent presentation software) slides with a collection of **static** pictures/ diagrams. A slightly better approach is to use a hardcoded presentation with animations that clearly demonstrates the steps of the algorithm. One drawback of this approach is that it is possible for the teacher to advance the animation too quickly. Also, it is difficult for the teacher to show another example that was not hardcoded or to demonstrate spontaneous examples asked by the students; the teacher has to either draw it directly on the PowerPoint slide manually (using the mouse or a stylus pen) or to draw the example on the board by hand (method 2).
2. The teacher hand-draws the examples on the board. Unlike PowerPoint where the teacher can advance to the next slide rapidly, this hand-drawing method allows the students to assimilate the steps taken by the algorithm in proper “human (student)

pace" as drawing successive steps of the algorithm may take some time. However, the drawback is that such manual drawing can be very time consuming if not executed/prepared properly. It is also error-prone if the algorithm is complex or if the teacher is in a rush. Students have to not only understand the algorithm but also copy the teacher's examples into their notes in order to review the illustrations later. The conflicting task to internalize the algorithm and copy the examples manually is exacerbated if the lesson is not recorded.

3. The teacher simply points the students to certain existing websites (usually created by independent developers) with some algorithm animations. Some of these websites are effective tools for teaching algorithms, as will be elaborated in Section 2 below. However, this method has drawbacks: most of these websites only support *a few* classical algorithm visualizations. Thus, if the teacher refers to one website for one algorithm and *different* website for *another*, the students may be confused by the different conventions and styles employed. The diversity of the various user interfaces adopted by these websites adds *another layer* of learning curve for the student. Some students have certain inertia that reduces their will to visit many different links. Also, this method *cannot be used* if the teacher is teaching a rare, non-classical algorithm that nobody has visualized before.

In our opinion, a better way to teach algorithms is to give a combination of a well-chosen and scripted example with animation (to introduce the concept *for the first time*) combined with two-or-three more examples on student-supplied input data. There are two conditions for this approach to be effective – the visualization of the spontaneous examples must be animated in a manner that is consistent and bug-free, and it must be done quickly and not take up too much time.

We made the decision to deliver our visualization on a web-based platform through the browser so that algorithmic knowledge is accessible to a larger pool of audience worldwide. Almost every devices nowadays – desktops, laptops, tablets, and even smartphones – have *built-in* web browser(s), e.g., Mozilla Firefox, Google Chrome, Safari[1], etc., that support HTML5 natively. There is thus no need to install any program or worry about cross-platform issue.

The choice of having interactive over static visualizations is for the students to gain more in-depth understanding about the algorithm being visualized. We believe that the students will have better understanding on how the algorithm works if they can enter the input data themselves.

With the rapidly increasing proliferation of portable devices with Internet connectivity: laptops, tablets, and smartphones[2], many students can access the web-based visualization during the lesson and follow the teacher's examples on their own devices. After class, students can then play around with the visualization again at their own pace/time to further reinforce their understanding of the algorithm's behavior.

[1]Safari is the default browser built-in on iPhone and iPad. This is worth mentioning because one of the features of our visualization is its accessibility on modern devices.

[2]As an illustration, during the fourth quarter of 2011, Apple sold 17.07 million iPhones (smartphone) and 11.12 million iPads (tablet), `www.apple.com/pr/library/2011/10/18Apple-Reports-Fourth-Quarter-Results.html`, last accessed: April 2012.

On the pedagogy side, having such visualizations will reduce the number of questions in quizzes and exams that simply test students' knowledge of how the algorithm will behave given certain input data. Such questions are now redundant as the students are expected to know how the algorithm works by using the visualization. Thus, the teacher can focus on testing on *more advanced usages* of the algorithms.

The choice of unified interface will also benefit the students. It is reasonable to expect a typical Computer Science class having more than one algorithm covered in the syllabus. By having a unified interface, students can expect similar look and feel when using different algorithm visualizations. Also, the students can just bookmark one webpage and use it for the *entire semester*. It is better than having a collection of different websites to visit. The student need not have to navigate through the lecture slides or handouts to get to the URLs recommended by the teacher. Every extra inconvenience increases the student's inertia and can be a possible cause for the student to be lazier and not bother to understand the algorithms properly.

This paper is organized as follows: In Section 2, we will detail and dissect the current available web-based algorithm visualizations for the algorithms listed in "Competitive Programming 2" (Halim and Halim, 2011). This competitive programming handbook is written by the first and the last authors of this paper. A substantial number of algorithms are discussed in that book, of which many do not have publicly available visualizations yet. At the same time, we also discuss the weaknesses of the existing visualizations through the lens of the objectives that we want to achieve, namely: provide visualizations for the non-classical algorithms, use unified interface, provide the ability to interactively enter user (student)-supplied input data, and provide the ability to access the visualization using modern web browsers particularly on smartphones. In Section 3, we discuss the details of our visualization project. In Section 4, we report the initial feedbacks from 31 students in National University of Singapore (NUS) who used the first version of the visualizations during semester 2 of academic year 2011/2012. In Section 5, we list down our preliminary conclusions regarding the effect of this visualization project. We conclude this paper by listing down our plans to continually improve this visualization project in the near future.

2. Existing Web-Based Algorithm Visualization

In this section, we summarize the results of our analysis of various algorithm visualization websites on the Internet as of April 2012. This report does not take into account algorithm visualizations that are not publicly available in the Internet. The list of algorithms is taken from "Competitive Programming 2" handbook (Halim and Halim, 2011). From our analysis, we found that there is a huge imbalance on the number of visualizations of classical – and usually easier – algorithms like sorting algorithms, binary search on a sorted array etc., versus the non classical and rare (usually harder) algorithms like Binary Indexed (Fenwick) Tree (Fenwick, 1994), finding Strongly Connected Components of a directed graph (Tarjan, 1972), computing the Maximum Flow of a network (Edmonds and Karp, 1972), etc.

2.1. *Standalone Visualizations*

We split our results into two categories: Standalone and Unified visualizations. We categorize the existing web-based algorithm visualizations in Table 1 as standalone visualizations because these websites only have a single to no more than three algorithm animations inside, i.e., their developers do *not* create visualizations for other algorithms. This causes an unnecessary learning curve when students have to learn different algorithms and have to visit different visualization websites. In order to make Table 1 concise, we only pick several representative visualizations per topic for the classic algorithm visualizations.

Table 1
Standalone visualizations (all URLs are correct as of April 2012)

Topic	Representative visualization examples	N	H	O
Various linear data structures	`apbrwww5.apsu.edu/myersb2/linked_list_animation.htm`		H	O
	`www.u-www.cosc.canterbury.ac.nz/mukundan/dsal/StackAppl.html`		H	O
Various sorting algorithms	`www.cs.auckland.ac.nz/~jmor159/PLDS210/Java/q_sort/tqs_new.html`		H	O
	`www.cse.iitk.ac.in/users/dsrkg/cs210/applets/sortingII/mergeSort/mergeSort.html`		O	
	`www.cs.oswego.edu/~mohammad/classes/csc241/samples/sort/Sort2-E.html`		H	O
Bitmask or bit manipulation	(see Section 4.1)	N		
Basic BST (Table)	`aleph0.clarku.edu/~achou/cs102/examples/bst_animation/BST-Example.html`			O
	`www.cs.jhu.edu/~goodrich/dsa/trees/btree.html`			O
	`www.csc.liv.ac.uk/~ullrich/COMP102/applets/bstree/`			O
Balanced BST (AVL)	`webdiis.unizar.es/asignaturas/EDA/AVLTree/avltree.html`			O
	`www.site.uottawa.ca/~stan/csi2514/applets/avl/BT.html`			O
	`www.cs.jhu.edu/~goodrich/dsa/trees/avltree.html`			O
Heap (priority queue)	`nova.umuc.edu/~jarc/idsv/lesson2.html`			O
	`www.cs.auckland.ac.nz/~jmor159/PLDS210/heaps.html`			O

To be continued

Continuation of Table 1

Topic	Representative visualization examples	N	H	O
Graph DS (Adj matrix or list)	(part of nova.umuc.edu/~ jarc/idsv/) (see Section 4.4)	N		
Union find disjoint sets	research.cs.vt.edu/AVresearch/UF/			O
	www.cs.unm.edu/~ rlpm/499/uf.html			O
Segment tree	(in our future works)	N		
Binary indexed (Fenwick) tree	(see Section 4.3)	N		
n-queens recursive backtracking	www.animatedrecursion.com/advanced/ the_eight_queens_problem.html		H	O
	yuval.bar-or.org/index.php?item=9		H	O
Classic dynamic programming	(in our future works)	N		
Graph traversal: BFS/DFS+	www.cs.sunysb.edu/~ skiena/combinatorica /animations/search.html		H	
	www.rci.rutgers.edu/~ cfs/472_html/ AI_SEARCH/SearchAnimations.html		H	
Finding cut vertices, bridges, SCCs	(see Section 4.5)	N		
MST (Prim's)	www.unf.edu/~ wkloster/foundations/ PrimApplet/PrimApplet.htm		H	O
	students.ceid.upatras.gr/~ papagel/ project/prim.htm		H	O
MST (Kruskal's)	www.unf.edu/~ wkloster/foundations/ KruskalApplet/KruskalApplet.htm		H	O
	students.ceid.upatras.gr/~ papagel/project/ kruskal.htm	H	O	
SSSP (Bellman Ford's)	(see Section 4.7)	N		
SSSP (Dijkstra's)	www.unf.edu/~ wkloster/foundations/ DijkstraApplet/DijkstraApplet.htm		H	O
	www.dgp.toronto.edu/~ jstewart/270/9798s/ Laffra/DijkstraApplet.html			O
APSP (Floyd Warshall's)	www.pms.ifi.lmu.de/lehre/compgeometry/ Gosper/shortest_path/shortest_path.html		H	O
Network flow (Edmonds Karp's)	felix-halim.net/research/maxflow/index.php		H	O
	www.eecs.wsu.edu/~cook/aa/lectures/ applets/ek/MaxFlowHome.html			
Algorithms on DAG	(in our future works, e.g., topological sort, shortest/longest path, counting paths)	N		
Algorithms on tree	(in our future works, e.g., shortest/longest path, all pairs shortest paths, Lowest Common Ancestor)	N		
Algorithms on bipartite graph	www-b2.is.tokushima-u.ac.jp/~ ikeda/ suuri/main/index.shtml		H	O

To be continued

Continuation of Table 1

Topic	Representative visualization examples	N	H	O
String matching	`cgjennings.ca/fjs/index.html`			O
Suffix tree	`illya-keeplearning.blogspot.com/2009/06/suffix-trees-java-applet.html`			O
Suffix array	`felix-halim.net/pg/suffix-array`			
Algorithms on polygon	(see Section 4.9)	N		
Convex hull (Graham's scan, Andrew's, Jarvis March)	`www.cs.princeton.edu/courses/archive/spr10/cos226/demo/ah/GrahamScan.html`		H	O
	`nms.lcs.mit.edu/∼aklmiu/6.838/convexhull/`			O
	`www.cs.unc.edu/∼snoeyink/demos/ch/Jarvis.html`			O

In the same table, we also highlight *three* possible potential issues of these *standalone* visualizations with respect to our objectives listed in Section 1 using the following codes:

1. **N** (not available): This algorithm is the rare one where Internet searches using relevant keywords do not yield public web-based visualization for it yet.
2. **H** (hardcoded, not interactive): This algorithm visualization is hardcoded (especially the ones that are just animated GIF), i.e., users cannot use their own input data to see how the algorithm being visualized will behave on the *new* input data.
3. **O** (old technology): This (older) algorithm visualization is using older technology (e.g., Flash, Java Applet) that is not truly portable as it cannot be accessed via modern smartphones (e.g., iPhones) or tablets (e.g., iPads).

2.2. *Unified Visualizations*

Other than the standalone visualizations listed in Table 1, we also found the following unified visualizations that offer *more than three* visualizations in their website:

1. `algoviz.org` is basically a portal that contains links to various algorithm visualization websites. In the future, our visualization project will also be listed there. However, as of April 2012, some of our algorithm visualizations in Section 3 (e.g., Fenwick Tree, Tarjan's SCC, etc.) are not yet available there.
2. `www.ansatt.hig.no/frodeh/algmet/animate.html` is another portal that compiles URLs to other visualization websites (similar nature with `algoviz.org`).
3. `nova.umuc.edu/∼jarc/idsv/` has visualizations for binary trees, several graph representation and algorithms, and sorting.
4. `www.csse.monash.edu.au/∼dwa/Animations/index.html` has several visualizations similar nova.umuc.edu/∼jarc/idsv/.

5. `research.cs.vt.edu/AVresearch/` has several algorithm visualizations developed between 2003 and 2009.
6. `www.cs.usfca.edu/~galles/visualization/Algorithms.html` is the closest unified visualization project that is the most similar with the ideas presented in this paper. It also built with HTML5. We highlight some of the differences between our visualization and this project in Section 3.

3. Our Unified and Interactive Web-Based Algorithm Visualization

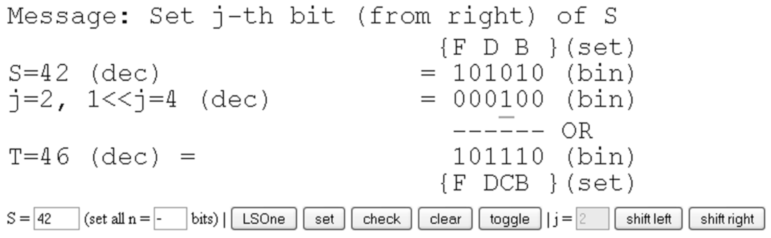
These are the algorithm visualizations that we have built as of April 2012. Our focus at the early stages of this visualization project is to provide visualizations of the non-classical algorithms rather than the more classical ones. We also highlight the differences and the extra features that we have incorporated for the algorithms that have been visualized by some other programmers and can be found publicly on Internet.

3.1. *Bitmask/Bit Manipulation/Bit String/Lightweight Small Set of Boolean*

Bit manipulation (bitmask) of integer values is usually a tricky subject for new Computer Science students. Therefore, having a bug-free visualization on bitmask like the one shown below will help both the teachers in showing various examples and the students in understanding the concept. For example, 42 | (1 « 2) = 46. This text-based visualization (see Fig. 1) will instantly show the necessary bit manipulations. We currently cannot find similar visualization in the Internet.

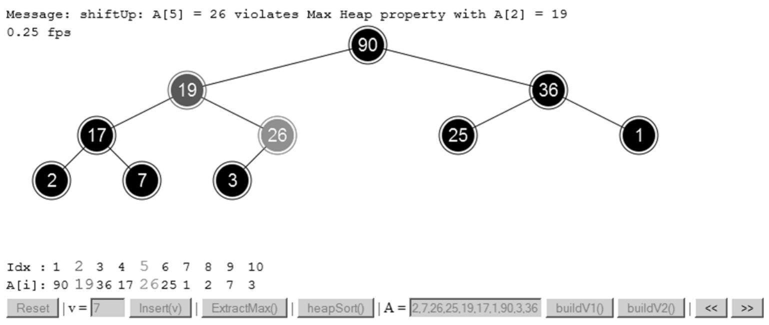
3.2. *Binary Heap Data Structure (Priority Queue)*

Binary Heap is a classical data structure commonly used to implement priority queue. There are (many) other similar visualizations in the Internet. The only strong point of having this visualization (Fig. 2) in our project is the similar unified look-and-feel as with the other visualizations in our project.



```
Message: Set j-th bit (from right) of S
                                {F D B }(set)
S=42 (dec)                     = 101010 (bin)
j=2, 1<<j=4 (dec)              = 000100 (bin)
                                 ------ OR
T=46 (dec) =                     101110 (bin)
                                {F DCB }(set)
```

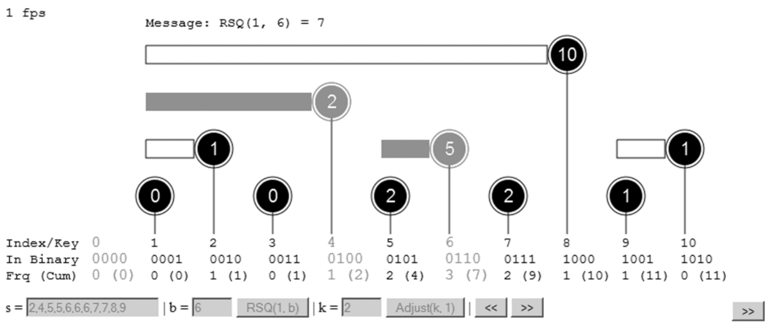
Fig. 1. Bitmask visualization.



```
Message: shiftUp: A[5] = 26 violates Max Heap property with A[2] = 19
0.25 fps
```

```
Idx : 1  2  3  4  5  6  7  8  9  10
A[i]: 90 19 36 17 26 25 1  2  7  3
```

Fig. 2. Binary heap visualization.



```
1 fps
Message: RSQ(1, 6) = 7
```

```
Index/Key  0      1      2      3      4      5      6      7      8       9       10
In Binary  0000   0001   0010   0011   0100   0101   0110   0111   1000    1001    1010
Frq (Cum)  0 (0)  0 (0)  1 (1)  0 (1)  1 (2)  2 (4)  3 (7)  2 (9)  1 (10)  1 (11)  0 (11)
```

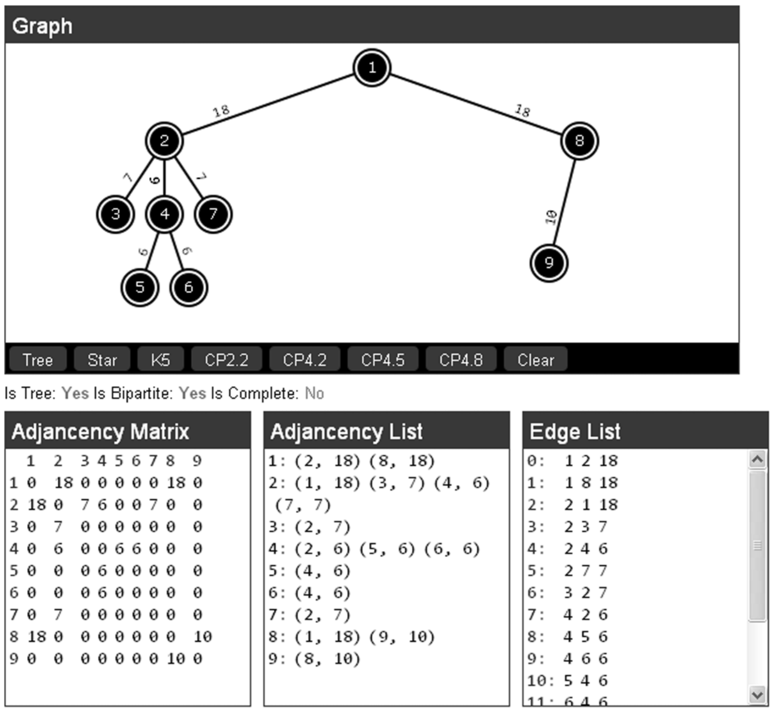
Fig. 3. Binary indexed (Fenwick) tree visualization.

3.3. *Binary Indexed Tree (BIT) or Also Known as Fenwick Tree*

BIT was invented by Fenwick (1994). This data structure is used to efficiently compute dynamic range sum queries (RSQ/cumulative frequency table). It has interesting usages in programming competitions. This data structure is included in the International Olympiad in Informatics (IOI) syllabus (Forišek, 2009), making it one of the studied data structure for IOI (high school) students. BIT operations can be cryptic for new users. For example, RSQ(1, **6**) is a summation of RSQ(1, **4**) + RSQ(5, **6**) because integer **6** ("1**1**0" in binary) becomes integer **4** ("1**0**0" in binary) if we strip off its least significant one. If we continue another step, integer **4** becomes integer **0** ("**0**00" in binary) and we stop here. This process is clearly shown and animated in Fig. 3. There is no existing BIT visualization that we can find on the Internet before ours.

3.4. *Graph Data Structure*

Graph visualizations are abundant in the Internet. However, almost every such visualizations uses *hardcoded* input graphs which limit its usage. We speculate that this is because it is hard to provide the input data for a graph and graph layout is another big subject by its own. Our visualization allows users to *draw their own* graph (Fig. 4). By having the users draw the graph themselves, we eliminate the need of using sophisticated graph



Is Tree: Yes Is Bipartite: Yes Is Complete: No

```
  1  2  3 4 5 6 7 8  9
1 0  18 0 0 0 0 0 18 0
2 18 0  7 6 0 0 7 0  0
3 0  7  0 0 0 0 0 0  0
4 0  6  0 0 6 6 0 0  0
5 0  0  0 6 0 0 0 0  0
6 0  0  0 6 0 0 0 0  0
7 0  7  0 0 0 0 0 0  0
8 18 0  0 0 0 0 0 0  10
9 0  0  0 0 0 0 0 10 0
```

```
1: (2, 18) (8, 18)
2: (1, 18) (3, 7) (4, 6)
 (7, 7)
3: (2, 7)
4: (2, 6) (5, 6) (6, 6)
5: (4, 6)
6: (4, 6)
7: (2, 7)
8: (1, 18) (9, 10)
9: (8, 10)
```

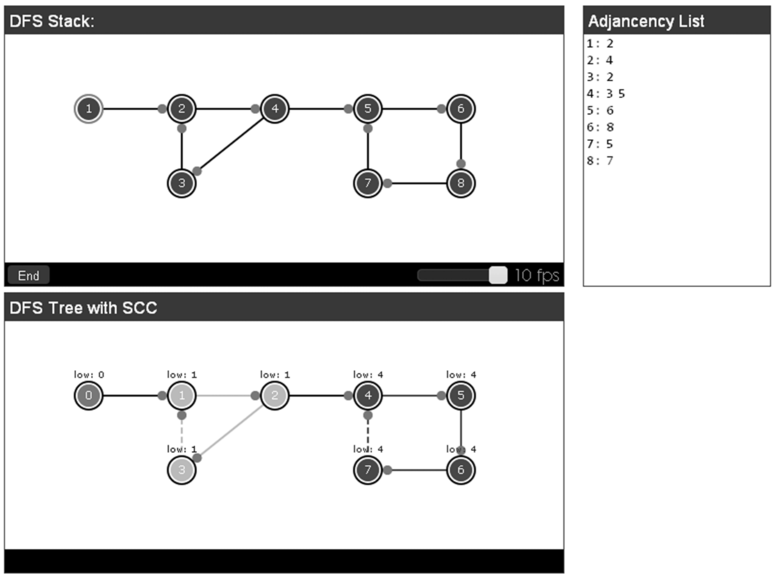
```
0:  1 2 18
1:  1 8 18
2:  2 1 18
3:  2 3 7
4:  2 4 6
5:  2 7 7
6:  3 2 7
7:  4 2 6
8:  4 5 6
9:  4 6 6
10: 5 4 6
11: 6 4 6
```

Fig. 4. Graph data structure visualization.

layout algorithm to layout the graph. Users can add/delete new vertex, add edges between vertices while simultaneously specify its direction and weight, and the visualization will be automatically updated with the correct Adjacency Matrix, Adjacency List, and Edge List representation of the user's input graph. As an additional feature, we also check if the graph is a tree, is a bipartite graph, or is a complete graph.

3.5. *Graph Traversal: Depth-First Search and Breadth-First Search*

In this visualization, users can draw their own graph as above and run one of two classical graph traversal algorithms (DFS or BFS). Many other graph traversal visualizations just stop at this point. However, we have incorporated extra features as these DFS and BFS algorithm can yield extra information about the graph structure. For DFS on undirected graph, we also compute vertices/edges that are classified as articulation points (cut vertices)/bridges. For DFS on directed graph, we also compute its Strongly Connected Components (SCCs) using Tarjan's (1972) algorithm as shown in the static figure blow (Fig. 5). Visualizations of these two DFS variants are currently not found elsewhere on the Internet. For BFS on unweighted graph, we also compute the shortest paths from starting vertex.



```
1: 2
2: 4
3: 2
4: 3 5
5: 6
6: 8
7: 5
8: 7
```

Fig. 5. Graph traversal visualization (showing the execution of Tarjan's SCC/DFS algorithm).

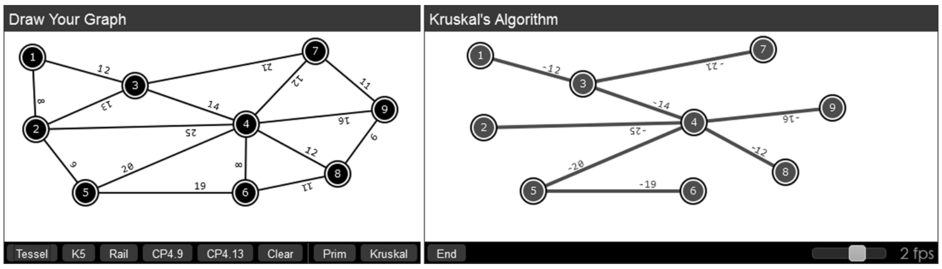


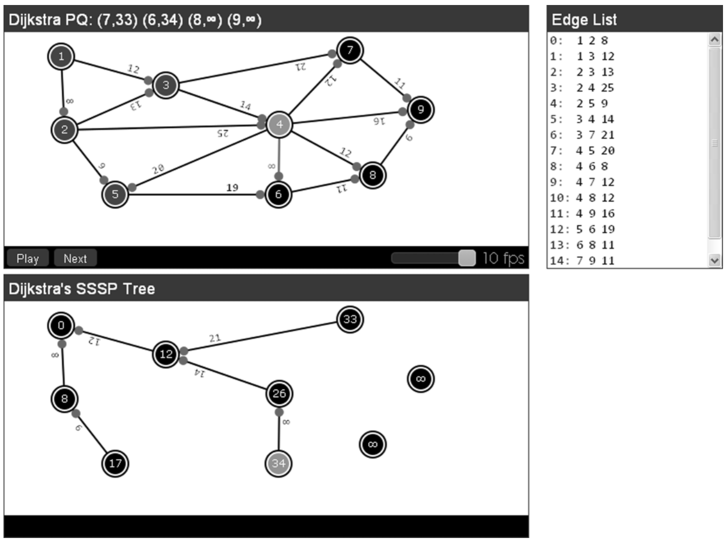
Fig. 6. MST visualization (showing the result of Kruskal's on input graph with all edge weights *negated*).

3.6. *Minimum Spanning Tree (MST): Prim's and Kruskal's*

There are several MST algorithm (Prim's and Kruskal's) visualizations on the Internet. The main strength of this visualization is that the users can draw their own graph (similar user interface as with the other visualizations above) and see how the classical Prim's or Kruskal's algorithm works on that graph. We added an extra feature of finding the *Maximum* Spanning Tree by simply negating all the edge weights (Fig. 6).

3.7. *Single Source Shortest Paths (SSSP): Dijkstra's and Bellman Ford's*

There are many other SSSP algorithm (especially Dijkstra's) visualizations on the Internet, although we have not managed to find the less frequently used Bellman Ford's algorithm visualization yet. This visualization maintains the same look and feel with the other visualizations in this project. It also inherits the same strength: Students can draw



```
Dijkstra PQ: (7,33) (6,34) (8,∞) (9,∞)
```

```
0:  1 2 8
1:  1 3 12
2:  2 3 13
3:  2 4 25
4:  2 5 9
5:  3 4 14
6:  3 7 21
7:  4 5 20
8:  4 6 8
9:  4 7 12
10: 4 8 12
11: 4 9 16
12: 5 6 19
13: 6 8 11
14: 7 9 11
```

Fig. 7. SSSP visualization (showing *intermediate* Dijsktra's algorithm execution with source = vertex 1).

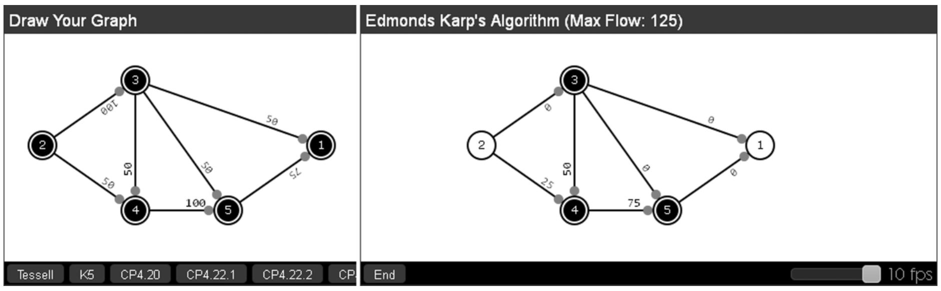


Fig. 8. Max flow visualization (showing the result of Edmond Karp's, source vertex 2, sink vertex 1).

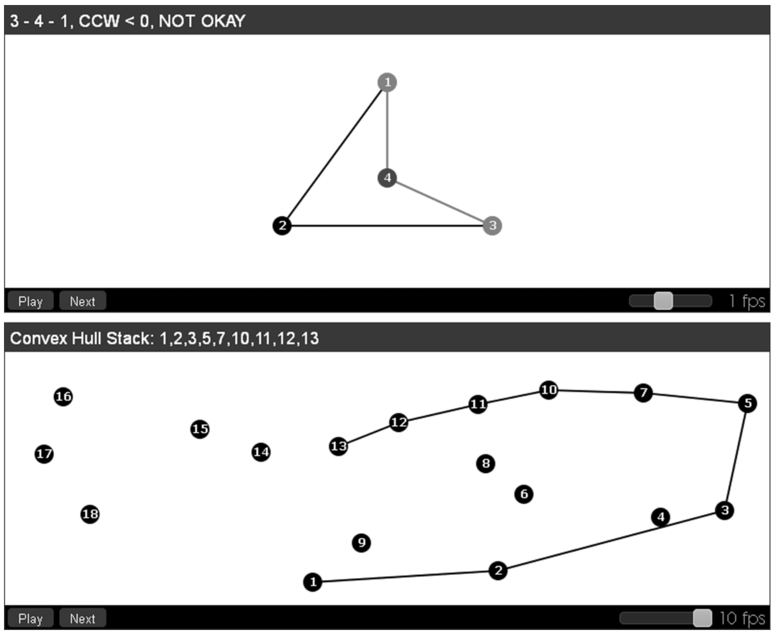
their own graph. We also provide the SSSP spanning tree for students to easily visualize the shortest path information.

3.8. *Maximum Flow on a Network: Edmonds Karp's*

Visualization for computing maximum flow on a network is rare. Our visualization (Fig. 8) maintains the same look and feel and graph drawing capability with the other visualizations in this project. In the near future (Section 5), we will add various network flow modeling and variants to enhance the strengths of this visualization.

3.9. *Algorithm on Polygon: IsConvex, ConvexHull*

There are several algorithm visualizations for finding the Convex Hull of a set of points in the Internet. However, there are mostly standalone visualizations. Our visualization sup-


3 - 4 - 1, CCW < 0, NOT OKAY

Convex Hull Stack: 1,2,3,5,7,10,11,12,13

Fig. 9. Intermediate computations of IsConvex() routine (top) and Graham's scan algorithm (below).

ports three Convex Hull algorithms: Graham's Scan (Graham, 1972) as shown in Fig 9. bottom, Jarvis's March (Gift Wrapping) algorithm (Jarvis, 1973), and Andrew's Monotone Chain algorithm (Andrew, 1979). With this feature, users can compare the behavior of these three algorithms on the same set of points.

Additionally, we can do several other things with a polygon, like an algorithm to test whether it is convex or concave (Fig. 9, top), computing the area and perimeter of an arbitrary polygon, checking if a point is inside or outside the given convex or concave polygon, or cutting a polygon with a straight line. Some of these features have already been implemented whereas the rest will be added in the near future (Section 5).

4. Students' Feedbacks

The visualizations in Section 3 were used for the first time in two modules offered in semester 2 (January–April 2012) at National University of Singapore:

1. CS3233 – Competitive Programming (24 students), taught by the first author. Visualizations used: bitmask, bit, representation, dfsbfs, mst, sssp, maxflow, and polygon/convexhull.
2. CS2020 – Data Structures and Algorithms Accelerated (42 students), taught by a colleague of the first author. Visualizations used: heap, representation, dfsbfs, mst, sssp.

To measure the subjective quality of these visualizations, we present the following survey to the students from these two classes. The survey was conducted in March 2012. At that time, all visualizations listed above have been shown to respective students. All 24 (100%) of CS3233 responded as these visualizations were used extensively in the first author's class. However, only 7 (16.7%) of CS2020 students responded as the visualizations were not used extensively in this module. We separate the results of these two student groups into two columns below. However, the general responses from the two groups are rather similar.

4.1. *Multiple Choice Questions (MCQ)*

Table 2
MCQ results

Multiple Choice Questions		CS3233 (24 responses)	CS2020 (7 responses)
1.	Does the visualization help you understand the algorithm taught *better*?	Yes (13/54.1%) No (0/0.0%) Neutral (11/45.8%)	Yes (5/71.4%) No (0/0.0%) Neutral (2/28.5%)
2.	When do you use the visualization? (select all that applicable to you)	In class (8/33.3%) After class (4/16.6%) Before test (14/58.3%) Others (6/25.0%)	In class (1/14.2%) After class (0/0.0%) Before test (5/71.4%) Others (3/42.9%)
3.	How do you access the visualization? (select all that applicable to you)	Desktops (14/58.3%) Laptops (14/58.3%) Tablets (3/12.5%) Smartphones (5/20.8%)	Desktops (4/57.1%) Laptops (6/85.7%) Tablets (0/0.0%) Smartphones (1/14.2%)

Several selected abridged reasons for question 1:

- **Yes**: Intuitive as I am a visual person; The visualizations help clarify concepts; I can play with various input values/scenarios; Useful to get initial feeling of how the algorithm works; I used to draw all these manually by myself.
- **Neutral**: Already understand the algorithm from the lecture/text book/reading the source code directly; Already know/master the algorithm beforehand; I prefer analysis (and proofs) rather than visualization to convince me that the algorithm works; Visualization bugs (in the version shown during the class) affect my understanding of the algorithm.

It is worth mentioning that none of the 31 respondents chose "No, the visualization does not help me understand more about the algorithm". The presence of these visualizations can thus be seen as a complementary pedagogical tool that will help a certain group of students immensely and will not have a negative effect on the other group of students.

From the second MCQ, we can see that some of the students indeed follow the discussions of the algorithm during the lesson. The most common usage pattern however is before tests/exams. This suggests that the primary usage of the visualizations is for self-learning and revision.

From the third MCQ, we can see that some students use their smartphones and/or tablets to access this visualization. While the number is still low at this point of time as not all respondents own these devices, we believe it will go up in the near future.

4.2. *Open Ended Essay Questions*

We also ask the students several open ended questions. We first ask them to list down the positive and negative aspects of having such algorithm visualizations. Some of the representative feedbacks are listed in Table 3.

We also ask them about their opinion on having a unified website that hosts not just the nine visualizations as listed in Section 3, but much more in the near future. Selected responses are listed below:

- **Positive**: It will be beneficial not only for this course (CS3233) but also other algorithm courses in NUS or other universities.
- **Positive**: A unified interface will be excellent. When students visualizing a certain algorithm in their mind or on paper, it is usually the same steps as shown in the visualizations.
- **Neutral**: To have them is a good idea. But if it does not come true, then it is not a big problem either.
- **Negative**: I personally prefer tracing algorithm by hand as it is easy to manipulate and help me remember the algorithm better, but I am sure there are others who prefer to have this tool.

5. Preliminary Conclusions, Future Works, and Acknowledgements

In this paper, we have presented a set of nine algorithm visualizations. They have a consistent look and feel, and are built using the same HTML5 canvas and Javascript code-base,

Table 3

Positive and negative aspects of having algorithm visualizations

Positive aspects	Negative aspects
The visualization always gives correct answer (after bug fixes).	Visualization bugs during the early rollout affect understanding.
Simple interface that is easy enough to use.	Some features are not user friendly yet (it still lacks proper documentation).
Ability to create usage scenarios on my own: Enter my own input values, draw my own graph, draw my own polygon.	It needs more than just nine algorithm visualizations as the first author teaches more algorithms in his CS3233 class.
Help me understand the algorithm better.	The presence of such visualizations may cause us not to practice on our own.
Faster than drawing by hand.	The steps shown need to be coupled with the highlight of algorithm (pseudo)codes.

and allows for interactivity with its users. More than half of the first 31 students who use these visualizations reported enhancement of their learning process whereas the other half reported neutral impact.

This visualization project is still an ongoing project. At the moment, this visualization project is hosted at: `www.comp.nus.edu.sg/~stevenha/visualization`. The URL may change in the future but the following keywords should help re-locating this visualization project: “algorithm”, “visualization”, “competitive programming”, “SoC”, “NUS”, and the author names.

In the near future, we have these plans:

1. Improve the features of existing visualizations: Add *Directed* MST (Arborescence) problem solvable with Chu-Liu/Edmonds’ algorithm (Chu and Liu, 1965; Edmonds, 1967), add various network flow modeling and variants (e.g., bipartite matching, min cost max flow, etc.), add several algorithm on polygons.
2. Add more classical algorithm visualizations to strengthen the library of algorithm visualizations that we currently have, e.g., algorithms on DAG (e.g., shortest/longest paths, counting paths), tree (e.g., Lowest Common Ancestor, etc.).
3. Add more non-classical algorithm visualizations that currently cannot be found elsewhere on the Internet, e.g., Segment Tree, Edmonds’ matching (Blossom Shrinking) algorithm (Edmonds, 1965), Hopcroft Karp’s bipartite matching algorithm (Hopcroft and Karp, 1973), Suffix Array and its Suffix Tree representation (Manber and Myers, 1991), etc.
4. Improve the user interface to make the visualizations more user-friendly (especially by adding source code highlights side by side with the visualization, adding the ‘step backwards’ feature) and add proper documentations.
5. Provide more pedagogical usage scenarios of the visualizations like having pop-quizzes on what will be the next step shown in the visualization, etc.
6. Make the visualization extensible to external developers by exposing our APIs.

We hope that our visualization project will be adopted by various Computer Science departments of various Universities worldwide.

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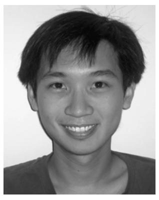

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