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Real Processes as Sources for Tasks in Informatics

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Abstract. Most of tasks in informatics are set with a background story about real processes even though the circumstances seem to be strained a little. At the same time natural sciences (physics, chemistry, biology, genetics, astronomy, geology, etc. . .) contain many interesting laws and facts, some of which can serve as a natural base for tasks in informatics. A survey of some of the laws, facts and techniques (especially ones of discretization), chosen to illustrate a composition of genuine tasks, are proposed in the paper.

Key words: tasks, informatics, real processes, sciences, physics, chemistry, biology, geology.

1. Survey of Ways to Generate Tasks and the Aim of Paper

Burton and Hiron (2008) offered two opposite ways to create a good task: “To wrap an abstract task inside a good story” and “To look around and to take inspiration from real things”. In Pankov (2008) we reviewed ways to generate task ideas based on “actors” and their actions in real and imaginary spaces and called such tasks *natural*. In other words, we have supposed that a good task should create a particular image in the mind of the contestant. In our experience, even questions using abstract mathematical spaces (Weeks, 1985) can be presented in a *natural* way in programming tasks. In Pankov and Baryshnikov (2009) we described some processes for taking any real (desirable) object (host country, city, university, sponsor, local sights, events, history, circumstances, etc. . .) and creating a task out of it for an informatics olympiad. The aim of this paper is to illustrate a possible involvement of scientific laws and facts into creating tasks in informatics but not to present ready-to-use tasks. Hence, we shall not give complete effective algorithms to all tasks.

We shall not consider the full procedure of creating a task with all of the corresponding restrictions, tests, etc. . . . It is not an aim of this paper and it was considered in details in Kemkes *et al.* (2007), Diks *et al.* (2008), Burton and Hiron (2008) and other publications. We shall give a brief description of “circumstances” or a background text of a task, which hopefully can be developed to full-grown.

We also shall not describe a sequence of tasks beginning from an evident one (with an algorithm to solve) and continuing to the more complex (where the problem of creating an appropriate algorithm arises).

In general, the following three types of tasks can be set if *the properties* of an object are given:

– Combinatory: how many *different* objects exist?

REMARK. The *difference* between objects is also an important notion. Often, the more objects are considered to be equivalent to each other the more difficult is the task (the more difficult is application of standard algorithms). And there is a lot of equivalence for *real* objects (rotation, reflection, translation and equivalence of atoms) which makes tasks more interesting.

– Optimization: find the maximum (or minimum) of the all objects.
– Interaction: detect the object by a minimal number or a restricted number of requests (trials). Such interaction can be implemented either as an interactive task (the contestant's program calls the jury's procedure) or as a simple (batch) task (see Task 12).

By our observation, most of tasks generated in such way are too difficult (NP-hard). On low level olympiads such tasks may be given with fewer restrictions, so they can be solved by full sorting.

On high level olympiads special restrictions can be put in place to distinguish an interesting algorithm.

For instance, in string processing, restrictions may be put on number of given words, on lengths of words, or on the number of symbols.

2. Physics

2.1. *Crystals*

The following type of task is classical. We mention it for completeness as related to the physical notion of a crystal. The full object (pattern) is composed of repeated sample (or "tile"). Given information about the pattern, find the sample (or "the least possible"/"smallest possible" sample) this pattern can be composed of.

Also, a two-dimensional rectangle net is usual. A two-dimensional triangle, a two-dimensional hexagonal and a three-dimensional rectangle nets also exist and can be involved in tasks. An interesting example of a crystal is a snowflake, which can be defined as a bounded figure with rotation symmetry of 60^0 and mirror symmetry with respect to an axis (and, consequently, to two other axes) passing through the center.

Task 1. Starting with a black-and-white photograph of a snowflake with a known center and horizontal axis of symmetry, only some black points and some white points remain. Given the coordinates (pairs of integer numbers) of these points in a rectangle system of coordinates with the angle XOY equal 60^0. Can these points belong to an ideal snowflake?

Formulas to solve: clockwise rotation 60^0 : $X_{\text{new}} = X + Y$; $Y_{\text{new}} = -X$. Reflection with respect to the X-axis: $X_{\text{new}} = X + Y$; $Y_{\text{new}} = -Y$.

The following task imitates similar processes of crystallization and of systematic life expansion.

Task 2. Given a net or a graph (a net is a particular case of graph) and a natural number K. If a vertex of a graph is marked now then all its neighbors are marked at the next step.

How many vertices ("centers of crystallization", "vegetative planting stocks", "ant colonies", etc...) must be marked initially to mark the entire graph in K steps?

Such a task is relatively simple. By involving more than one type of crystals in the same media or more than one of (competing) species one can yield more variations and more of a challenge in a task.

2.2. *Conservation Laws*

Law of conservation of mass and of linear momentum:

There are some (massive pointwise) objects moving along a straight line. If two or more objects are too close and they clash then the operating person can slightly deviate some of them. Those which are not deviated, merge together. The velocity of the new/whole object is calculated using the law of conservation of linear momentum: if the objects had masses $M_1, \dots, M_k$ and velocities $V_1, \dots, V_k$ (positive or negative) then the velocity of the merged object is

$$V := (M_1 * V_1 + \cdots + M_k * V_k)/(M_1 + \cdots + M_k).$$

Task 3. Given number N of objects, (positive integer) mass $M[i]$, (integer) initial position $X[i]$ and (integer) initial velocity $V[i]$ of all objects, $i = 1 \dots N$; $X[1] < \cdots < X[N]$.

Find the smallest possible absolute value of velocity of the merged object. Due to conditions of the task, the output is a rational number. So, it must be presented as <integer>/<natural> (as a fraction in its simplest form) where the HCF of the numerator and the denominator must equal 1.

EXAMPLE. $N = 3$, $X[1] = 10$, $X[2] = 20$, $X[3] = 30$, $M[1] = 100$, $X[2] = 500$, $X[3] = 104$, $V[1] = 7$, $V[2] = 20$, $V[3] = -7$. Answer: $7/51$ [mass of the merged object is 204].

Law of the conservation of the electrical charge.

Task 4. Given a graph, its V vertices are charged electrically (given integer non-zero numbers $C[i]$, $i = 1 \dots V]$) and its arcs of given lengths $L[i, j]$ (natural numbers) are nonconductors, $i, j = 1 \dots V$. Also, there is a given length $L1$ (natural number) of (conducting) wire. If the charged objects are connected with the conductor wire then their common charge is the sum of the all charges.

Find A) the smallest possible absolute value or B) the greatest possible absolute value of charge of any part of the graph which can be obtained by cutting the wire into pieces and connecting some vertices (objects) with these pieces along the arcs.

EXAMPLE. $V = 3$, $C[1] = 10$, $X[2] = -20$, $X[3] = -12$, $L[1, 2] = L[1, 3] = L[2, 3] = 100$, $L1 = 103$. Answer A: 2. Answer B: 32.

2.3. *Methods of Physical Investigation*

The process of balancing (with or without weights) is a source for many tasks.

Task 5. There is a balance scale and N objects of weights (kg) $W[1], \ldots, W[N]$ (given natural numbers). If difference between sides on the balance is greater than K kg then the scales turn upside down (K is a given non-negative integer). A robot can carry and put only one object on the scales at any given moment.

What is the minimum number of robots necessary to put all N objects on the scales?

EXAMPLE. $N = 6$, $W[1] = 1$, $W[2] = 8$, $W[3] = 5$, $W[4] = 20$, $W[5] = 20$, $W[6] = 1$, $K = 1$. Answer: 4 robots [in two steps].

Task 6. There are the scales and N weights (kg) $W[1], \ldots, W[N]$ (natural numbers) and an object of unknown weight $W1$. It is known that $1 \leqslant W1 \leqslant W0$; $W0$ is a given natural number; the HCF of $W[1], \ldots, W[N]$ is 1.

What is the minimum number of consecutive weightings necessary to detect $W1$ or to make the conclusion that it is impossible if the weights can be put A) on one of the scales? B) on both scales? (The three possible responses: the left scale is heavier; balance; the right scale is heavier).

EXAMPLE (a standard set of weights). $N = 4$, $W[1] = 1$, $W[2] = 2$, $W[3] = 2$, $W[4] = 5$, $W0 = 10$. Answer A): 3 weightings [$W[1]$ is not necessary].

Detecting particles. $N \times N$ square detectors form a big square. Particles fly above in straight lines. A particle flying across a detector (including its sides and vertices) sometimes activates it. A detector can be activated by more than one particle at the time. In such case, the precise number of particles crossing the detector is not detected by it.

Task 7. Given is an integer N and the list of activated detectors. Find the smallest possible number of particles which could activate these detectors.

EXAMPLE. $N = 5$, activated detectors: (1,1); (1,4); (5,1); (5,5). Answer: 2.

REMARK. Such tasks must be solved with integer numbers (with vulgar fractions). If division of numbers is used then there can arise mistakes because of rounding error.

3. Chemistry

3.1. *Chemical Formulas*

In this section we will not consider *real* chemical elements and molecules with their concrete properties; we will consider mathematical tasks arising in chemistry. Thus, we will use convenient denotations looking like chemical ones. Denote (conventional) chemical elements with capital Latin letters, so the greatest possible number of elements does not exceed 26 (this is not essential). Let a number of atoms of each element in a molecule be written after the denotation of the element. Elements in a formula will be written in the alphabetical order.

So, the molecule of (conventional) water H_2O may be written as $A2K1$ or $P1Q2$.

Task 8. Given chemical formulas and the number of atoms of each element in these formulas find the greatest number of *molecules* of these types which can be made of these atoms.

EXAMPLE.

Input: Two formulas: $A5B1$; $B3C2$; three elements: A 20; B 10; C 3.

Output: 5.

$[4(A5B1); 1(B3C2)]$.

REMARK. If the mixture of these chemical substances is a gas then each molecule occupies same volume of space (in the initial approximation). So, the condition of the task is natural: find the greatest volume occupied.

Task 9. Given: chemical formulae before a reaction and possible chemical formulae after the reaction. Can such reaction exist from the mathematical standpoint of view? All listed chemicals must be involved. If it can, then find the smallest possible coefficients (natural numbers) in the formulae to make balance of chemicals before and after the reaction.

Solving. Obviously, this task is reduced to a system of linear homogeneous Diophantine equations (a positive solution is to be found). If two equations contain the same unknown then it can be excluded. So, we obtain one or more equations with different unknowns. Moving backward we obtain the solution if it exists. Outlines of the algorithm are seen as follows.

EXAMPLE 1 (iron oxidation).

Input: (before): $F1$, $O2$; (after) $F2O3$.

Output: Yes; $4 * F1 + 3 * O2 = 2 * F2O3$.

Solving of Example 1. The task is $X_1 * F1 + X_2 * O2 = X_3 * F2O3$, the system is $X_1 = 2 * X_3, 2 * X_2 = 3 * X_3$.

The first equation has the general solution $X_1 = 2 * T_1$, $X_3 = T_1$. Substituting we obtain: $2 * X_2 = 3 * T_1$, hence $T_1 = 2 * T_2$, $X_2 = 3 * T_2$; $X_1 = 4 * T_2$; $X_3 = 2 * T_2$. Taking the least possible value $T_2 = 1$, we obtain a solution.

REMARK. In real (simple) tasks the following algorithm is also valid. Choose one of the coefficients as 1. If all other coefficients are defined uniquely then they are rational numbers. Multiplying all coefficients by the LCM of all denominators we obtain the required natural coefficients.

EXAMPLE 2 (conventional).

Input: (before): $C2D3, B2C5, B3D2$; (after) $B3C3D3, C2D1$.

Output: Yes; $4 * C2D3 + 3 * B2C5 + 5 * B3D2 = 7 * B3C3D3 + 1 * C2D1$.

3.2. *Structural Chemical Formulae – Chemical Graph Theory*

In addition to a chemical formula, it is possible that bonds between atoms (or valence of each atom) are given. These bonds define a set of graphs. This set can be used to compose various other tasks.

Task 10. Given a chemical formula (atoms are vertices of a graph) and A) valences (number of arcs from each vertex) of all atoms or B) a list of atoms which each atom must be connected with. How many sufficiently different graphs (i.e. different chemicals) exist?

EXAMPLE A) (isopentans).

Input: formula $C5H12$; valence of C (carbon) is 4; valence of H (hydrogen) is 1.

Answer: 3 [five C atoms are connected with the following arcs: 1) 1–2, 2–3, 3–4, 4–5; 2) 1–2, 2–3, 3–4, 3–5; 3) 1–2, 1–3, 1–4, 1–5].

3.3. *Chemical Reactions*

There are M known chemicals; the first N of them are present.

Some of the chemicals can be obtained from other ones (we will only consider reactions that cause two chemicals to become one).

All reactions are given as four numbers $B1, B2, A$ (all different natural numbers in $1 \ldots M$) and T (integer number denoting releasing heat, if $T > 0$, or required heat, if $T < 0$) indicating the A-th chemical is obtained of $B1$-th and $B2$-th ones.

Task 11. Find the most profitable way to obtain the given $A0$-th chemical of the list $M - N + 1 \ldots M$ (if it is possible), i.e., such sequence of reactions $B1, B2, A, T$ that:

– all As are in $M - N + 1 \ldots M$ and different; the last A is $A0$;

– each $B1, B2$ are either in $1 \ldots N$ or of preceding As;

– all As except $A0$ are used in following reactions;

– the sum of all Ts has the greatest possible value.

3.4. *Methods of Chemical Analysis*

Task 12. Given the list of T trial chemicals, the list of U chemicals to be detected and the list of the results $R[i, j]$ (encoded as natural numbers in $[1 \ldots K]$, K is a given natural number too) of reactions of i-th chemical of the first list with the j-th chemical of the second list, $i = 1 \ldots N, j = 1 \ldots M$.

Find the smallest possible number of reactions to detect the unknown chemical from the second list (if it is possible).

4. Genetics

4.1. *Reading Genetic Code*

The following situation is a classic example. We recall it for completeness.

Reading genetic code. There are many identical chromosomes and the task is to get to know the sequence of "letters" written on given chromosome. To read all the information on a long chromosome is too difficult, so chromosomes are split into pieces which are sufficiently small to be read easily. The splits are random.

Task 13. Given a set of words. Find the smallest possible length of a word containing all the given words as sub-words.

Task 14. Given a set of words. How many words of given length containing all the given words as sub-words exist? (If such words do not exist then output 0).

4.2. *Detecting Number of Chromosomes*

If two hereditary characters, defined by genes, are positioned on the same chromosome, then they are inherited simultaneously. Every gene of a child coincides with the corresponding gene of one of its parents. Suppose that some genes are of phenotype character (can be observed or detected in any way).

Task 15. Given the natural number $N > 2$ and set of M triples of words of length N: $P1$, $P2$, B fulfilling the condition: each letter $B[k] = P1[k]$ or $B[k] = P2[k]$, $k = 1 \dots N$.

Find the minimal number of subsets in a decomposition of the set $1 \dots N$ such that for every subset S and every triple $P1, P2, B$ the intersection $B \cap S = P1 \cap S$ or $B \cap S = P2 \cap S$.

REMARK. The researcher does not know the number of chromosomes and order of coding, so they have arranged ascertained characters arbitrarily.

EXAMPLE.

$N = 5$; $M = 2$;

$(TEWPT, EDWBV, TEWPV)$; $(DEWXT, EFWBT, EEWBT)$.

Answer: 3 [the first set: $\{1, 4\}$; the second set: $\{2, 3\}$; the third set: $\{5\}$].

The answer is the lower boundary for number of chromosomes of this species.

Solving of this task is relatively simple but demands fluency in treating sets.

5. Geology

Restoring the chronological sequence of sedimentary layers (of geological epochs).

Millennium after millennium, sedimentary layers are deposited onto the sea bed. Each epoch deposits its own layer. Due to the different geological environments, some layers are absent in some parts of the sea bed. Researchers have taken samples in different places, and have separated each sample into layers, denoting all ascertained layers with letters.

Task 16. Given a set S of words. Each letter can be in each word only once; if one letter precedes other one, in any word, then the same must occur in other words too. Find a (long) word W containing these letters only (each letter only once) such that all given words can be obtained from it by erasing some letters. Is such word unique?

EXAMPLE.
Input: MTUG; TGH; TFH.
Output: not unique [three possible words: MTFUGH; MTUFGH; MTUGFH].
Solution. Probably, the following algorithm is the best.
Denote a set of non-empty words in S as S'. Firstly, $S' = S$.
$W = \mathrm{empty} - \mathrm{word}$; while S' is not empty {compare the first letters in all words in S': if there is the only one preceding to others then concatenate it to W and avoid it from all words in S' else {output "not unique" and stop}}; output W.

This task is relatively simple. But subsequent development in the area could turn some of the strata upside down. And then the following task arises.

Task 17. In conditions of Task 16, some (less than half of) words can be reversed. The quest is same.

6. Astronomy

There is a Sun in the center of a solar system and N Planets rotating around the Sun in circular orbits. When all inner Planets will overshadow Sun from the standpoint of the outermost (Nth) Planet simultaneously?

Choose the following measure of angles: the full rotation (of 2π radian) is equal to 1.

Task 18. Given natural $N > 1$, initial angles of all planets $A[1], \dots, A[N]$ (as rational numbers between 0 and 1), periods of rotation of all planets $P[1] < \cdots < P[N]$ (as positive rational numbers and the maximal absolute values of angles of overshadowing $M[1], \dots, M[N-1]$ (as (small) positive rational numbers). Find (if it is possible) the minimum number T (a rational number) such that at the moment T from initial moment the following inequalities will be primarily true:

abs(angle Nth-Planet – the-center-of-Sun – Kth-Planet) $\leqslant M[K]$,
for all $K = 1 \dots N-1$.

REMARK. Actually, the planes containing orbits of different planets and of satellites, often do not coincide (eclipses of the Moon and of the Sun do not occur every month). But taking it into account only results in complicating the task too much, because the lines of intersection of these planes (so-called lines of nodes) constantly change.

7. Conclusion

We hope that this paper will promote the idea of involving laws, ideas and methods of sciences into informatics olympiads, making them more engaging for young people, and attracting contestants' attention to vast applications of informatics. Accordingly, this preposition can also inspire a greater interest of young people in learning sciences and perhaps even in helping to make an appropriate career choice in future.

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