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From Al-Khwarizmi to Algorithm

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Mohammad ibn Musa al-Khwarizmi (780–850), Latinized as *Algoritmi*, was a Persian mathematician, astronomer, and geographer during the Abbasid Caliphate, a scholar in the House of Wisdom in Baghdad.

In the 12th century, Latin translations of his work on the Indian numerals introduced the decimal number system to the Western world. Al-Khwarizmi's *The Compendious Book on Calculation by Completion and Balancing* resented the first systematic solution of linear and quadratic equations in Arabic. He is often considered one of the fathers of algebra.

Some words reflect the importance of al-Khwarizmi's contributions to mathematics. "Algebra" (Fig. 1) is derived from *al-jabr*, one of the two operations he used to solve quadratic equations. *Algorism* and *algorithm* stem from *Algoritmi*, the Latin form of his name.

Fig. 1. A page from al-Khwarizmi "Algebra".

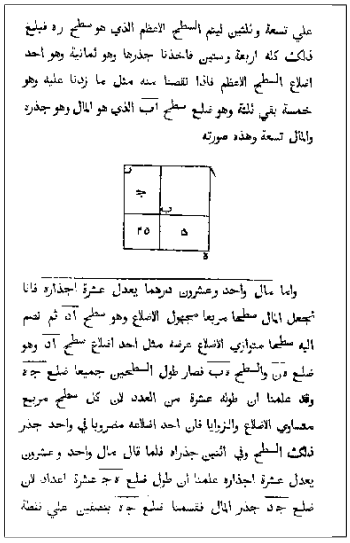
Few details of al-Khwarizmi's life are known with certainty. He was born in a Persian family and Ibn al-Nadim gives his birthplace as Khwarazm in Greater Khorasan. Ibn al-Nadim's *Kitāb al-Fihrist* includes a short biography on al-Khwarizmi together with a list of the books he wrote. Al-Khwārizmī accomplished most of his work in the period between 813 and 833 at the House of Wisdom in Baghdad.

Al-Khwarizmi contributions to mathematics, geography, astronomy, and cartography established the basis for innovation in algebra and trigonometry. His systematic approach to solving linear and quadratic equation led to *algebra*, a word derived from the title of his 830 book on the subject: "The Compendious Book on Calculation by Completion and Balancing".

On the Calculation with Hindu Numerals written about 825, was principally responsible for spreading the Hindu-Arabic numeral system throughout the Middle East and Europe. It was translated into Latin as *Algoritmi de numero Indorum*. Al-Khwarizmi, rendered as (Latin) *Algoritmi*, led to the term "algorithm".

Some of his work was based on Persian and Babylonian astronomy, Indian numbers, and Greek mathematics. When, in the 12th century, his works spread to Europe through Latin translations, it had a profound impact on the advance of mathematics in Europe.

The Compendious Book on Calculation by Completion and Balancing (*al-Kitab al-mukhtaṣar fi ḥisab al-jabr wal-muqabala*) is a mathematical book written approximately 830 CE (Fig. 2 a). The book was written with the encouragement of Caliph al-Ma'mun as a popular work on calculation and is replete with examples and applications to a wide range of problems in trade, surveying and legal inheritance. The term "algebra" is derived from the name of one of the basic operations with equations (al-jabr, meaning



a)

the first quadrate, which is the square, and the two quadrangles on its sides, which are the ten roots, make together thirty-nine. In order to complete the great quadrate, there wants only a square of five multiplied by five, or twenty-five. This we add to thirty-nine, in order to complete the great square S H. The sum is sixty-four. We extract its root, eight, which is one of the sides of the great quadrangle. By subtracting from this the same quantity which we have before added, namely five, we obtain three as the remainder. This is the side of the quadrangle A B, which represents the square; it is the root of this square, and the square itself is nine. This is the figure:—

Demonstration of the Case: "a Square and twenty-one Dirhems are equal to ten Roots."

We represent the square by a quadrate A D, the length of whose side we do not know. To this we join a parallelogram, the breadth of which is equal to one of the sides of the quadrate A D, such as the side H N. This parallelogram is H B. The length of the two

b)

Fig. 2. The *Book of Algebra* by Al-Khwarizmi.
a) The original Arabic print manuscript of the *Book of Algebra* by Al-Khwarizmi.
b) A page from *The Algebra of Al-Khwarizmi* by Fredrick Rosen, in English.

"restoration", referring to adding a number to both sides of the equation to consolidate or cancel terms) described in this book. The book was translated in Latin as *Liber algebrae et almucabala* by Robert of Chester in 1145. A unique Arabic copy is kept at Oxford and was translated in 1831 by F. Rosen (Fig. 2 b). A Latin translation is kept in Cambridge It provided an exhaustive account of solving polynomial equations up to the second degree, and discussed the fundamental methods of "reduction" and "balancing", referring to the transposition of terms to the other side of an equation, that is, the cancellation of like terms on opposite sides of the equation.

Al-Khwarizmi's method of solving linear and quadratic equations worked by first reducing the equation to one of six standard forms (where b and c are positive integers):

- Squares equal roots ($ax^2 = bx$).
- Squares equal number ($ax^2 = c$).
- Roots equal number ($bx = c$).
- Squares and roots equal number ($ax^2 + bx = c$).
- Squares and number equal roots ($ax^2 + c = bx$).
- Roots and number equal squares ($bx + c = ax^2$).

By dividing out the coefficient of the square and using the two operations *al-jabr* ("restoring" or "completion") and *al-muqabala* ("balancing"). Al-jabr is the process of removing negative units, roots and squares from the equation by adding the same quantity to each side. For example, $x^2 = 40x - 4x^2$ is reduced to $5x^2 = 40x$. Al-muqabala is the process of bringing quantities of the same type to the same side of the equation. For example, $x^2 + 14 = x + 5$ is reduced to $x^2 + 9 = x$.

The above discussion uses modern mathematical notation for the types of problems which the book discusses. However, in al-Khwarizmi's day, most of this notation had not yet been invented, so he had to use ordinary text to present problems and their solutions.

For example, for one problem he writes, (from an 1831 translation):

> *If someone say: "You divide ten into two parts: multiply the one by itself; it will be equal to the other taken eighty-one times." Computation: You say, ten less thing, multiplied by itself, is a hundred plus a square less twenty things, and this is equal to eighty-one things. Separate the twenty things from a hundred and a square, and add them to eighty-one. It will then be a hundred plus a square, which is equal to a hundred and one roots. Halve the roots; the moiety is fifty and a half. Multiply this by itself, it is two thousand five hundred and fifty and a quarter. Subtract from this one hundred; the remainder is two thousand four hundred and fifty and a quarter. Extract the root from this; it is forty-nine and a half. Subtract this from the moiety of the roots, which is fifty and a half. There remains one, and this is one of the two parts.*

Etymologically, the word algorithm is a combination of the Latin word *algorismus*, named after Al-Khwarizmi, and the Greek word *arithmos*, meaning "number". In Eng-

lish, it was first used in about 1230 and then by Chaucer in 1391. English adopted the French term, but it wasn't until the late 19th century that "algorithm" took on the meaning that it has in the modern mathematics and finally in the 20th century it became a common word in computer science.

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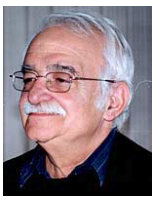

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